Inside the TWFE Coefficient

The Four Algebraic Decompositions of CBS Table 1

A teaching deck on what your regression is actually computing

Lu & Yu (2015) reported $\hat{\beta}^{\textsf{twfe}} = -0.322$

A single number from a real paper.

What is this number actually computing?

That is the only question this deck answers.

By the end, you will compute this weight by hand

Today's single, narrow goal:

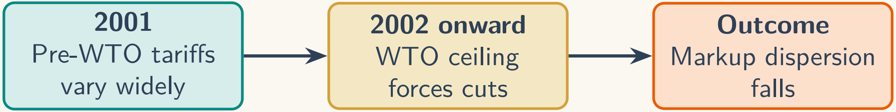
understand *how* the TWFE coefficient averages dose-specific effects — the pure mechanics of the weights, nothing about causal inference.

No ATT, no ACRT, no SPT, no identification.
Just the algebra of four weight formulas.

Act I — The Setup

Where the number comes from

China's WTO accession hit 155 industries with different tariff cuts



2001
Pre-WTO tariffs vary widely

2002 onward
WTO ceiling forces cuts

Outcome
Markup dispersion falls

Lu & Yu ran a TWFE regression on the industry-year panel.
Our question: what does that regression actually compute?

One coefficient β^{twfe} carries the entire dose response

$$y_{it} \;=\; \alpha_i \;+\; \beta^{\text{twfe}} \cdot D_i \cdot \text{Post}_t \;+\; \lambda_t \;+\; \varepsilon_{it}$$

y_{it}: outcome

α_i: industry FE

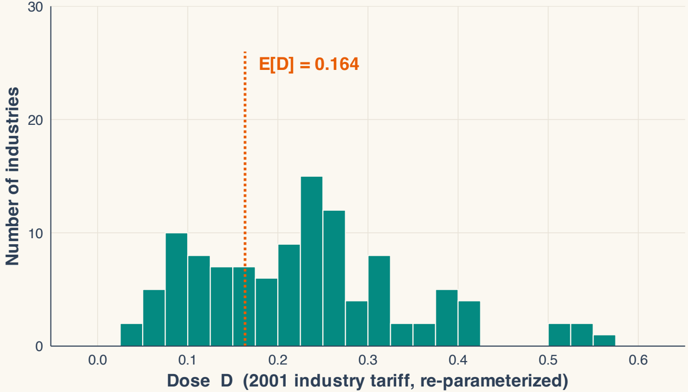
D_i: dose (pre-WTO tariff)

Post$_t$: 1 if $t \geq 2002$

λ_t: year FE

β^{twfe}: the number we want to decompose

D_i is one continuous number per industry — and there are 155 of them



Simulated to match Lu–Yu: mean ≈ 0.16*, right-skewed, 28% at dose 0.*

Every weight formula is a function of *this* distribution

The four weight formulas in Table 1

use nothing but the dose distribution.

If you know $E[D]$, Var(D), $f_D(l)$, $P(D \geq l)$, $E[D \mid D \geq l]$, and d_L —

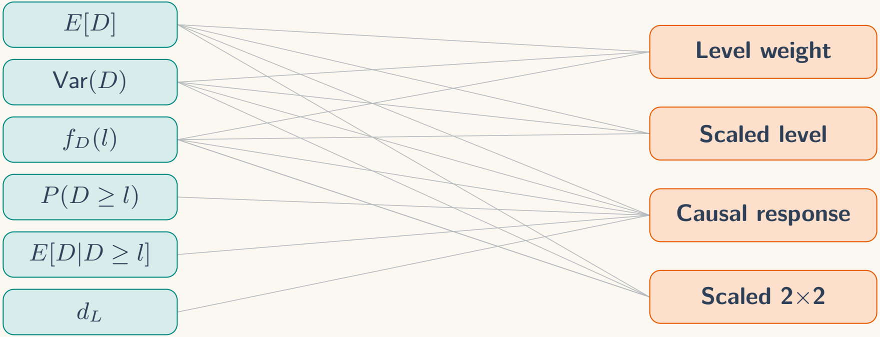
you can compute every weight in Table 1 yourself.

No regression output needed. These are population quantities of D.

Act II — Six Ingredients, Four Formulas

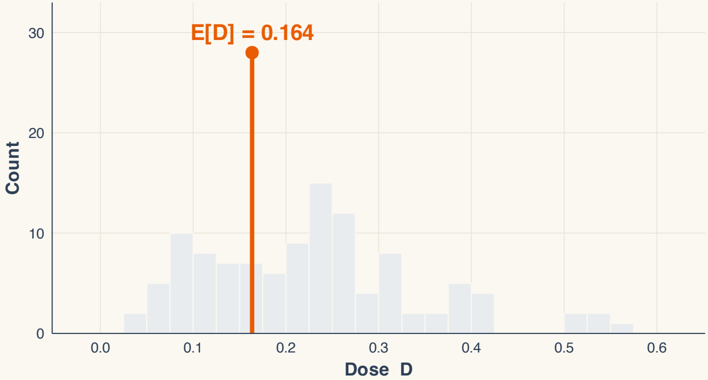
The moving parts

Six ingredients combine into four formulas



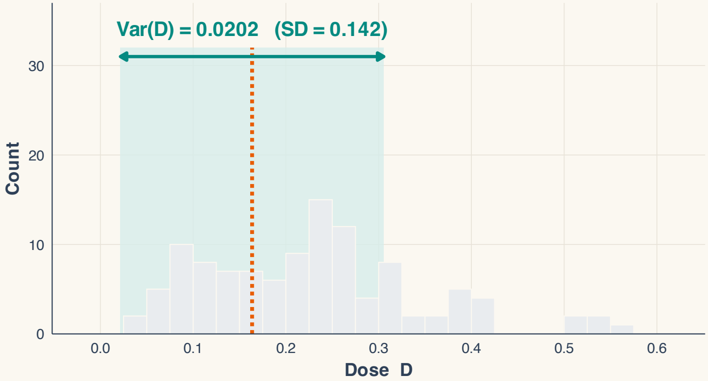
The same six quantities, recombined four ways.

Ingredient 1: the mean dose $E[D]$ anchors every formula



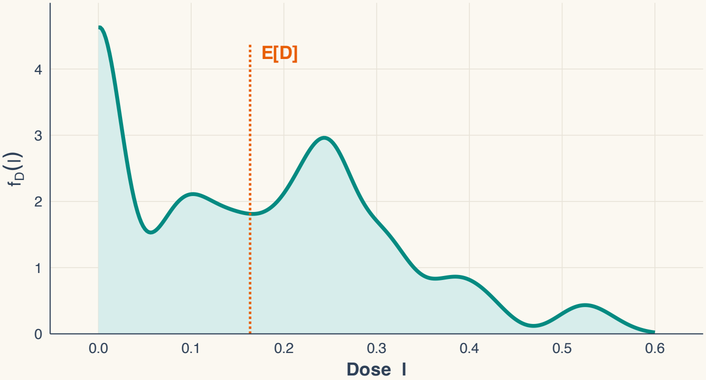
$E[D] = 0.164$ — the dose an “average” industry experienced.

Ingredient 2: the variance Var(D) sets the scale



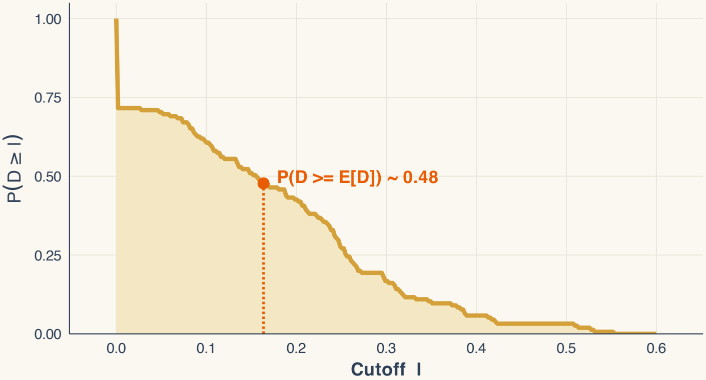
Var$(D) = 0.020$ — it appears in the *denominator* of every weight formula.

Ingredient 3: the density $f_D(l)$ says "where are the industries?"



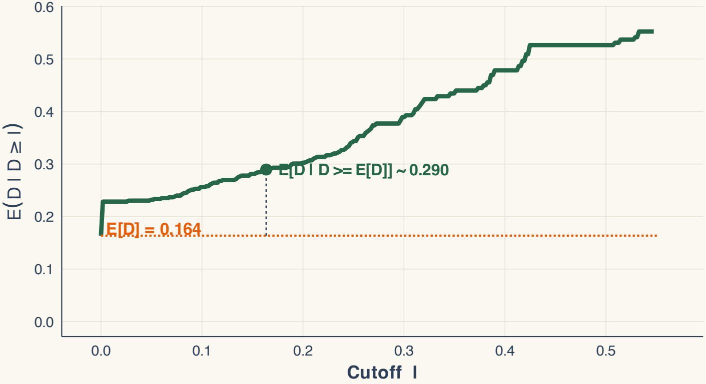
High where many industries cluster. Low in the tails.

Ingredient 4: the survival $P(D \geq l)$ says "how many above the cutoff?"



At $l = 0$ it's 1. At $l = d_U$ it's 0. Monotone decreasing.

Ingredient 5: the conditional mean $E[D \mid D \geq l]$ tracks the upper tail



Starts at $E[D]$ when $l = 0$. Climbs to d_U as $l \to d_U$.

Ingredient 6: the minimum positive dose d_L closes the list

The smallest positive dose in the sample.

$$\boxed{d_L = 0.027}$$

Only appears in the boundary term of the causal-response decomposition.
Included for completeness — not the star of any slide.

The First Decomposition

Level weights — where sign flips happen

Imagine β^{twfe} as a weighted average of *dose-level effects*

Each dose l has some "effect at that dose."
What weights does the regression put on those effects?

$$\boxed{\beta^{\mathsf{twfe}} = \int w^{\mathsf{lev}}(l) \cdot (\text{effect at } l) \; dl}$$

The question of this section: what is $w^{\mathsf{lev}}(l)$, concretely?

Compute $w^{\text{lev}}(0.10)$ by hand for a 10%-tariff industry

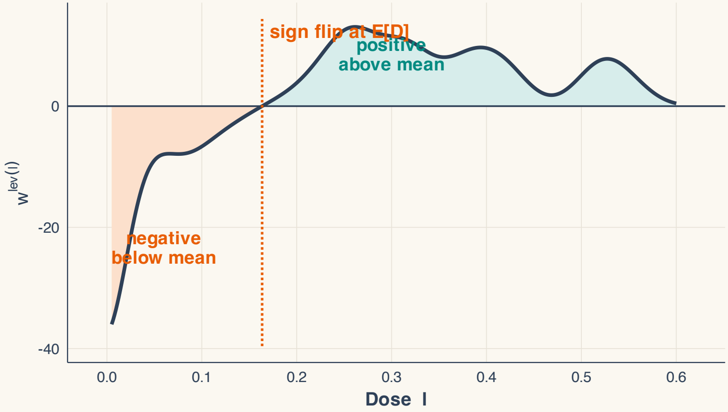
$$w^{\text{lev}}(0.10) \;=\; \frac{(0.10 - \boxed{0.164}\,) \cdot \boxed{f_D(0.10)}}{\boxed{0.020}}$$

Plug in the numbers:

$$w^{\text{lev}}(0.10) \;=\; \frac{(-0.064) \cdot 2.07}{0.020} \;=\; \mathbf{-6.65}$$

A negative weight, just because 0.10 is below $E[D] = 0.164$.

$w^{\text{lev}}(l)$ flips sign at $l = E[D]$



Compute $w^{\text{lev}}(l)$ in R with three lines

```
# Ingredients from 00_setup.R: dose, E_D, Var_D, f_D()

w_lev <- function(l) (l - E_D) * f_D(l) / Var_D

# At l = 0.10:
w_lev(0.10)
#> [1] -6.653
```

That is the whole formula. Three ingredients, one division.

The level weight: negative below the mean, positive above

$$w^{\text{lev}}(l) = \frac{(l - E[D]) \cdot f_D(l)}{\text{Var}(D)}$$

sign-carrier
negative if $l < E[D]$

mass-weight
how many industries

normalizer
scale by variance

Integrates to zero. Not an average.

The Second Decomposition

Scaled level weights — same flip, new scale

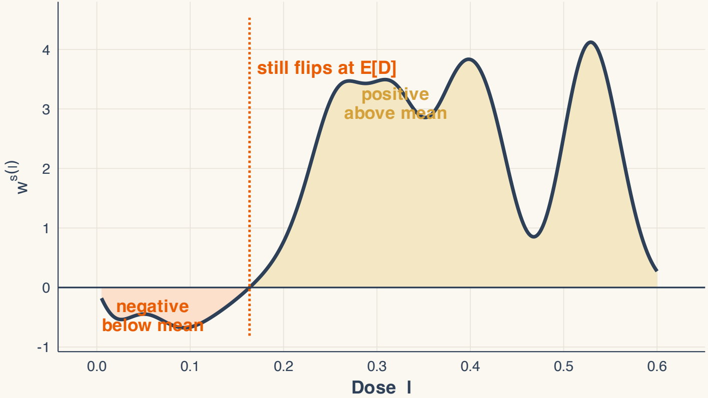
Multiply by l: the sign flip refuses to go away

$$w^s(l) = \boxed{l} \cdot \frac{(l - E[D]) \cdot f_D(l)}{\mathsf{Var}(D)}$$

Multiply the level weight by the dose itself.

Think: "per-unit-dose level effect" instead of "per-dose level effect."

$w^s(l)$ still flips at $E[D]$ — just weighted by the dose



$$w^s(0.10) = 0.10 \cdot (-6.65) = \mathbf{-0.665}$$

Compute $w^s(l)$: one multiplication on top of the level formula

```r
w_s <- function(l) l * (l - E_D) * f_D(l) / Var_D

# At l = 0.10:
w_s(0.10)
#> [1] -0.665
```

The only change from the level weight is the leading l.

The scaled level: now it integrates to 1 — but the sign flip remains

$$w^s(l) = l \cdot \frac{(l - E[D]) \cdot f_D(l)}{\mathrm{Var}(D)}$$

Differences from level weight:

- Weights now integrate to **1**, not zero.
- Still negative below $E[D]$ and positive above.
- Low-dose industries still get penalized.

The Third Decomposition

Causal-response weights — the “nice-looking” one

$w^{\textsf{acrt}}$ weights are non-negative and integrate to 1

Non-negative. Integrates to 1. Looks like an honest average.

But “non-negative and integrates to 1”
does **not** mean “the dose density.”
We will see exactly how they differ.

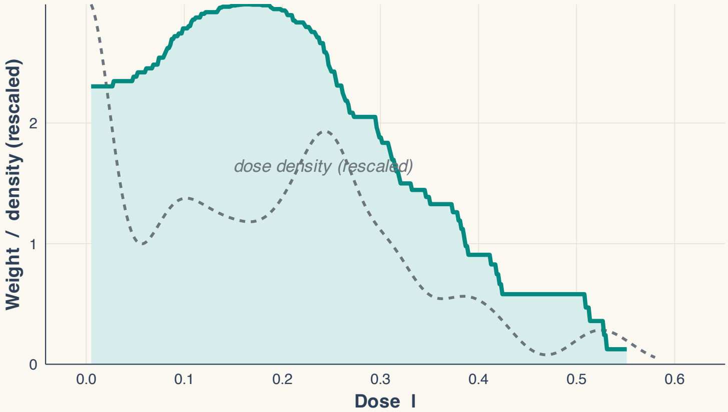
Compute $w^{\text{acrt}}(0.10)$ by hand

$$w^{\text{acrt}}(0.10) \;=\; \frac{\big(\, E[D \mid D \geq 0.10] \;-\; E[D] \,\big) \cdot P(D \geq 0.10)}{\text{Var}(D)}$$

$$w^{\text{acrt}}(0.10) \;=\; \frac{(0.257 - 0.164) \cdot 0.600}{0.020} \;=\; \mathbf{2.79}$$

A positive weight — in fact every $w^{\text{acrt}}(l)$ is non-negative.

$w^{\text{acrt}}(l)$ is positive everywhere — but it is not the density



Compute $w^{\text{acrt}}(l)$ in R

```r
w_acrt <- function(l) {
  (E_D_given_ge(l) - E_D) * P_D_ge(l) / Var_D
}

# At l = 0.10:
w_acrt(0.10)
#> [1] 2.785
```

Conditional mean, survival, variance. No sign flip possible.

The causal-response weight: positive, but the wrong shape

$$w^{\text{acrt}}(l) = \frac{\boxed{(E[D \mid D \geq l] - E[D])} \cdot \boxed{P(D \geq l)}}{\boxed{\mathsf{Var}(D)}}$$

non-negative factor
$E[D \mid D \geq l] \geq E[D]$

survival
$\in [0, 1]$

normalizer

Non-negative. Integrates to 1. Not the dose density.

The Fourth Decomposition

Scaled 2×2 weights — over pairs of doses

The last decomposition integrates over *pairs* of doses

Not a weight on single doses. A weight on **pairs** (l, h) with $h > l$.

$$\boxed{\beta^{\text{twfe}} = \iint_{h>l} w^{2\times 2}(l,h) \cdot (\text{2-group effect})\; dh\, dl}$$

Think "every pair of industries, compared to each other."

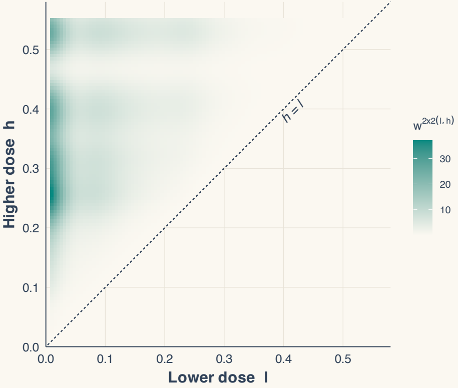
Compute $w^{2\times2}(0.05, 0.25)$ for a low-high pair

$$w^{2\times2}(l,h) = \frac{(h-l)^2 \cdot f_D(h)\cdot f_D(l)}{\mathsf{Var}(D)}$$

$$w^{2\times2}(0.05, 0.25) = \frac{(0.20)^2 \cdot 2.0 \cdot 1.6}{0.020} \approx \mathbf{6.4}$$

Quadratic in the gap $(h - l)^2$ — pairs that are far apart get much more weight.

Far-apart pairs get the most weight — the $(h-l)^2$ shows up visually



The scaled 2×2 weight: quadratic scaling on dose gaps

$$w^{2\times 2}(l,h) = \frac{(h-l)^2 \cdot f_D(h) \cdot f_D(l)}{\mathrm{Var}(D)}$$

quadratic gap
heavily rewards distance

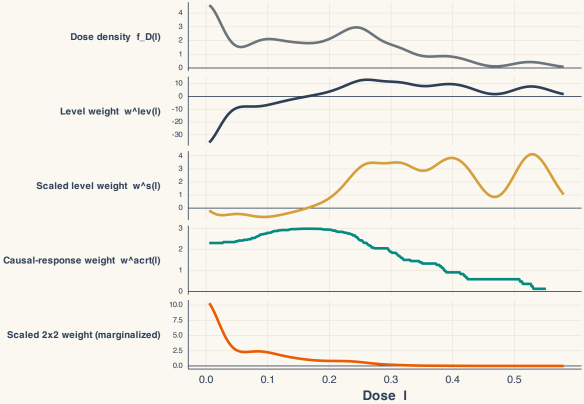
joint density
both doses must be common

Non-negative. Integrates to 1. Biased toward extreme pairs.

Act III — Putting It Together

Four weightings, one distribution

The same distribution, weighted four radically different ways



Four formulas, four different stories

Level

$$w^{\mathsf{lev}}(l) = \frac{(l - E[D])\, f_D(l)}{\mathsf{Var}(D)}$$

negative below mean, sums to zero

Scaled level

$$w^{s}(l) = \frac{l\,(l - E[D])\, f_D(l)}{\mathsf{Var}(D)}$$

still flips sign, sums to 1

Causal response

$$w^{\mathsf{acrt}}(l) = \frac{(E[D|D \geq l] - E[D])\, P(D \geq l)}{\mathsf{Var}(D)}$$

non-negative, but not the density

Scaled 2×2

$$w^{2\times 2}(l, h) = \frac{(h - l)^2\, f_D(h)\, f_D(l)}{\mathsf{Var}(D)}$$

over pairs, favors extremes

$\beta^{\textbf{twfe}}$ **is a weighted integral.**

And now you can compute the weights yourself.

Six ingredients. Four formulas. One dose distribution.