

Gov 2001: Problem Set 1

Spring 2026

Instructions:

- The Problem set is due on **February 3, 11:59 PM Eastern Time**.
- Please upload a PDF of your solutions to Gradescope. Make sure to assign to each question all the pages with your work on that question.
- **Do not use AI assistants (ChatGPT, Claude, Copilot, etc.) on this problem set.** Work with each other instead. The struggle is where learning happens.
- Remember: 70% of your grade comes from in-class exams. Use problem sets to *learn*, not just to get answers.

Survey

How many hours (roughly) did you spend on Gov 2001 (other than lectures and sections) in the past week?

Short Questions

1. For a sample space $\Omega = \{A, B, C, D\}$, check whether these sets are event spaces: (1) $S_1 = \{\{A, B\}, C, D\}$; (2) $S_2 = \{\emptyset, A, \Omega\}$; (3) $S_3 = \{\emptyset, A, \{B, C, D\}, \Omega\}$.
2. For a countable infinite sample space $\Omega = \{A_1, A_2, A_3, \dots\}$, is it possible that all singleton events have equal probability, i.e. $P(A_i) = P(A_j)$ for $\forall i, j$? Use the axioms of probability to justify your answer.
3. Let A, B, C be events. Prove that $P(A \cap (B \cup C)) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$ directly using the axioms of probability (in particular, finite additivity).

Long Questions

4. A polling firm surveys 1,200 likely voters in a swing state before the 2024 election. They record party registration and candidate preference:

	Harris	Trump	Undecided	Total
Democrat	336	18	30	384
Republican	24	372	36	432
Independent	114	168	102	384
Total	474	558	168	1,200

- (a) Calculate $\mathbb{P}(\text{Trump} \mid \text{Republican})$. Interpret this probability in one sentence.
- (b) Calculate $\mathbb{P}(\text{Republican} \mid \text{Trump})$. Interpret this probability in one sentence.

(c) A cable news pundit says: “86% of Trump voters are Republicans, so if you meet a Republican, they’re almost certainly a Trump voter.” Evaluate this reasoning using your answers from (a) and (b). What error is the pundit making?

5. A political scientist studies voter turnout in a state with three types of counties:

County Type	Share of Registered Voters	Turnout Rate
Urban	50%	58%
Suburban	30%	72%
Rural	20%	64%

(a) Using the Law of Total Probability, calculate the overall turnout rate for the state.

(b) A voter is selected at random from those who voted. What is the probability they are from a suburban county? (Use Bayes’ Rule.)

(d) **Simpson’s Paradox:** Suppose in a subsequent election:

- Turnout *increases* in every county (Urban: 62%, Suburban: 75%, Rural: 68%)
- But the population shifts toward urban areas (Urban: 60%, Suburban: 25%, Rural: 15%)

Calculate the new overall turnout rate. Is it possible for overall turnout to *decrease* even though every county’s turnout increased? Explain what’s happening.